ASCHAM SCHOOL

YEAR 12
MATHEMATICS EXTENSION 1
TRIAL EXAMINATION 2014

GENERAL INSTRUCTIONS

5 minutes reading time.
Working time 2 hours.
Use black or blue pen.
A table of standard integrals is provided on the back page.
Approved calculators and templates may be used.

Total Marks - 70

Section 1 – MULTIPLE CHOICE (1 mark each)

- Attempt Questions 1-10.
- Allow approximately 15 minutes.
- Answers on the separate sheet provided.
- Write your name/BOS number, teacher's name.

Section 2– Question 11 – 14 (15 marks each)

- Allow 1 hour 45 minutes.
- Start each question in a new booklet.
- If you use a second booklet for a question, place it inside the first.
- Write your name/BOS number, teacher's initials and question number on each booklet.

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Section 1 **Multiple choice** **(1 mark each)**

(Mark the correct answer on the sheet provided)

1. The tan of the angle between the two lines $2x - y = 4$ and $y = -4x$ is

(A) $\frac{2}{9}$ (B) $\frac{6}{-7}$ (C) $\frac{-2}{9}$ (D) $\frac{6}{7}$

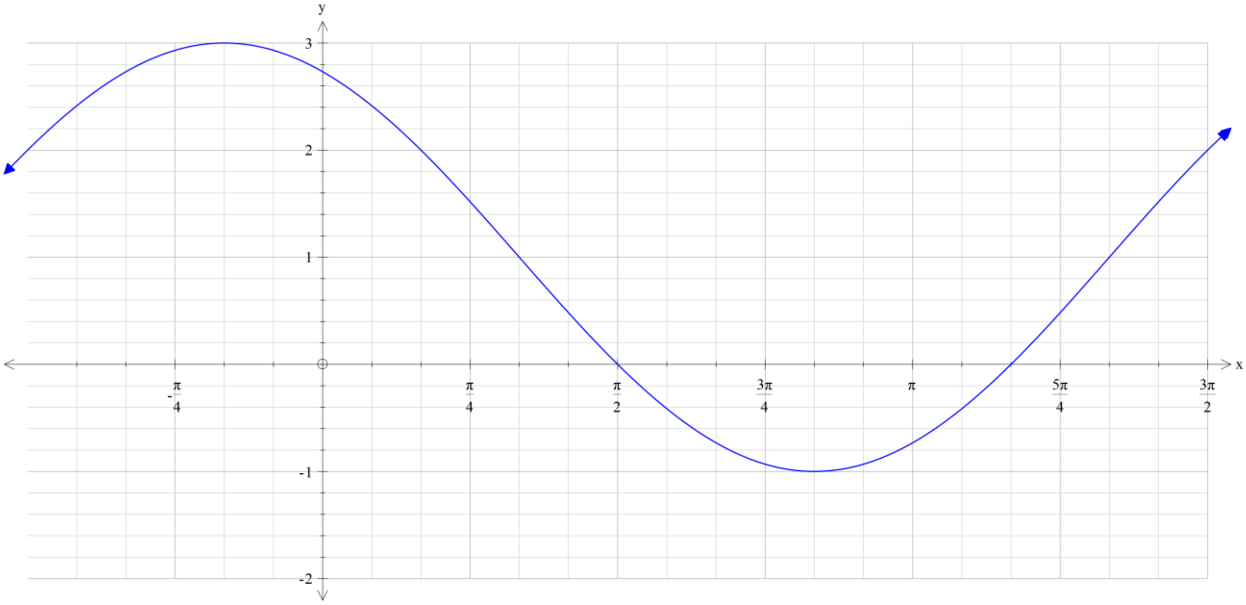
2. The point that divides the interval joining A(-2, 3) to B(5, 4) externally in the ratio of 2: 3 is,

(A) $\left(\frac{4}{5}, 3\frac{2}{5}\right)$ (B) $\left(3\frac{1}{5}, -\frac{1}{5}\right)$ (C) $(-16, 1)$ (D) $(19, 21)$

3. $\frac{d}{dx}(\sec x)$ is

(A) $\sqrt{1+\tan^2 x}$ (B) $\sec x \tan x$ (C) $-\operatorname{cosec} x \cot x$ (D) $\tan^2 x$

4. The equation of the graph below is:



(A) $y = 2\cos(x + \frac{\pi}{6}) + 1$ (B) $y = 2\cos 4(x - \frac{\pi}{6}) + 1$

(C) $y = 4\sin 2(x - \frac{\pi}{12}) - 1$ (D) $y = 3\cos(2x + \frac{\pi}{6}) - 1$

5. Solve the inequality $\frac{-2}{x-3} \leq 1$

(A) $x \leq 1$ or $x \geq 3$

(B) $1 \leq x \leq 3$

(C) $x = 3$ or 1

(D) $x \leq 1$ or $x > 3$,

6. Which of the following is **not** true about the function $y = \left|x^2 - 9\right| + 2$?

(A) The graph is continuous everywhere

(B) $f(-3) = 2$

(C) $f(x) \geq 2$ for all values of x

(D) $f'(x) = 2x$ for all $x > 0$

7. The acceleration of a particle moving in a straight line is given by $\ddot{x} = -4x - 16$, where its displacement from a fixed point O is x m. The motion is simple harmonic. What is the centre of the motion and the period?

(A) centre $= -4$ and period $= \pi$

(B) centre $= 4$ and period $= \pi$

(C) centre $= -2$ and period $= \pi$

(D) centre $= 2$ and period $= 2\pi$

8. How many solutions does the equation $\sin 2\theta = \cos\theta$ have in the domain $0 \leq \theta \leq 2\pi$

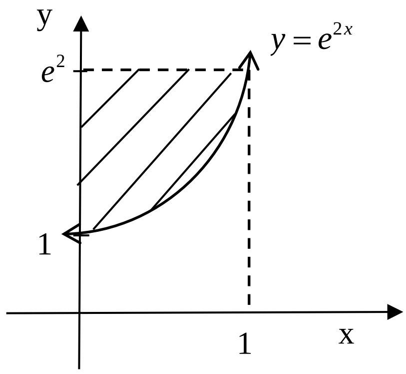
(A) 2

(B) 3

(C) 4

(D) 5

9.



To find the area of the shaded region in the diagram shown, four different students proposed the following calculations.

i) $\int_0^1 e^{2x}dx$ ii) $e^2 - \int_0^1 e^{2x}dx$ iii) $\int_1^{e^2} e^{2y}dy$ iv) $\int_1^{e^2} \frac{\ln x}{2}dx$

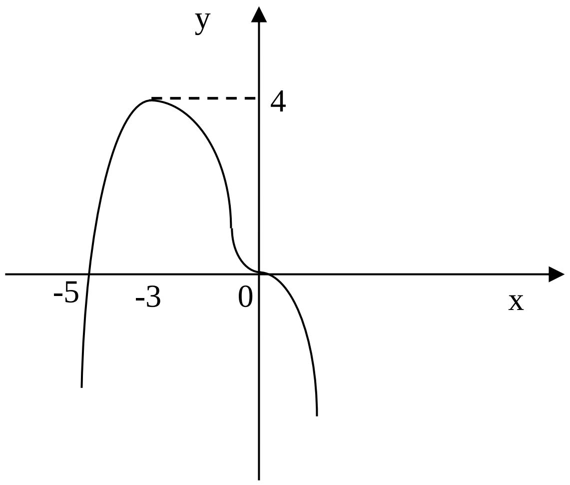
Which of the following is correct?

(A) ii) only   (B) ii) and iii) only

(C) i) ii) iii) and iv)   (D) ii) and iv) only

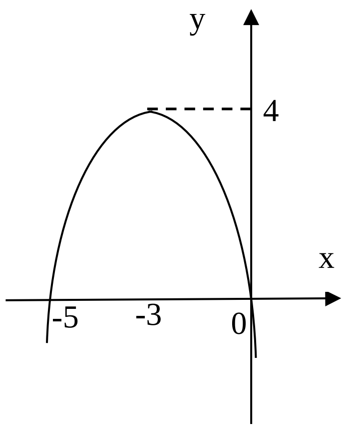
Turn over to the last multiple choice question

10. The graph of the function $y = f(x)$ is shown below

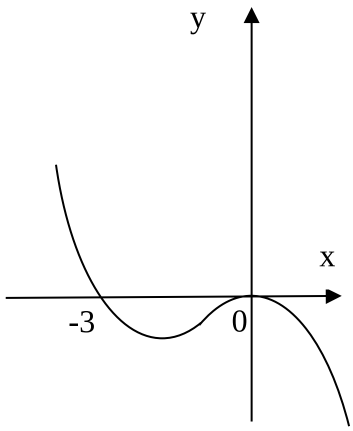


Which of the following could be the graph of the derivative function $y = f'(x)$?

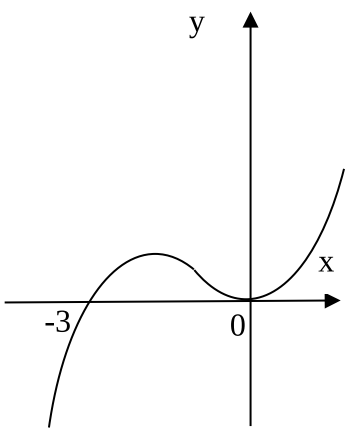
(A)



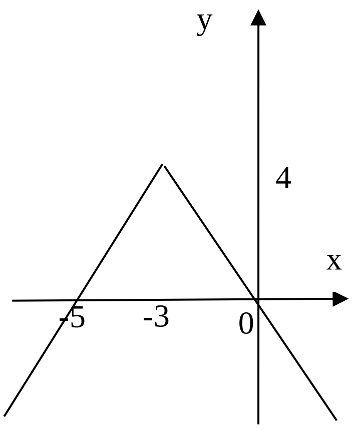
(B)



(C)



(D)



End of Multiple choice. Question 11 begins on the next page

Question 11 **Begin and label a new booklet.** **15 marks**

a) Differentiate with respect to x

(i) $\cos^{-1}(3x)$ [1]

(ii) $\tan^2 3x$ [2]

b) Evaluate $\int_0^3 y\sqrt{y+1}\,dy$ using the substitution u = y+1 [2]

c) α,β,γ are the roots of $2x^3 - x^2 + 5x + 2 = 0$
Evaluate $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ [2]

d) Find the general solution of $\tan(x - \frac{\pi}{3}) = \sqrt{3}$ [1]

e) Prove by induction that $5^n - 3^n$ is even if n is a positive integer. [4]

f) The volume of water in a hemispherical bowl of radius 10cm is given by

$$v = \frac{\pi}{3}x^2(30 - x)$$

where x cm is the depth of the water at any time *t*. The bowl is being filled at a constant rate of 3π cm^3/ min. At what rate is the depth increasing when the depth is 5cm. [3]

Question 12 **Begin and label a new booklet.** **15 marks**

a) (i) Sketch the graph of $y = \dfrac{x^2 - 1}{x^2 + 1}$ [2]

Hence find the value(s) of k such that $k = \dfrac{x^2 - 1}{x^2 + 1}$ has

(ii) 1 solution [1]

(iii) 2 solutions [1]

(iv) 0 solutions [1]

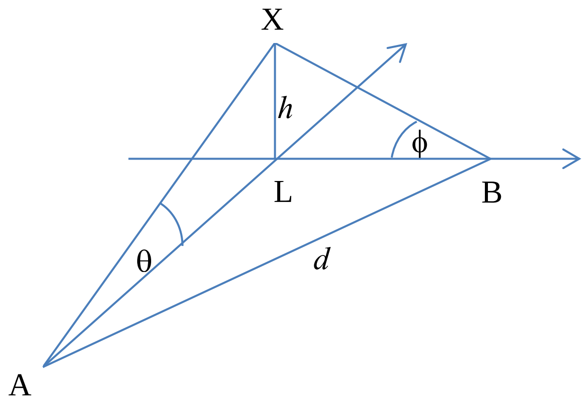
b) (i) State the domain and range of $y = 3\sin^{-1}(1 - x)$ [2]

(ii) Find the gradient of $y = 3\sin^{-1}(1 - x)$ when $x = 1$ [2]

(iii) Sketch $y = 3\sin^{-1}(1 - x)$ [2]

Question 12 continues on the next page.

c) The angle of elevation to the top of a lighthouse, X, from a ship A due south of it is θ. From a ship B due east of the lighthouse the angle of elevation to the top of the lighthouse is ϕ. The distance between the ships is d metres.



(i) Show that the height of the top of the lighthouse above sea level is given by

$$h = \frac{d \tan\theta \tan\phi}{\sqrt{\tan^2\theta + \tan^2\phi}}$$ [3]

(ii) If a chart shows that the top of the lighthouse is 115m above sea level and the ships' captains measure the angles of elevation to be 18° and 23°15', find the distance between the ships. [1]

Question 13 Begin and label a new booklet. 15 marks

a) A vessel is being filled at a variable rate and the volume of liquid in the vessel at any time t is given by $V = A\left(1 - e^{-kt}\right)$

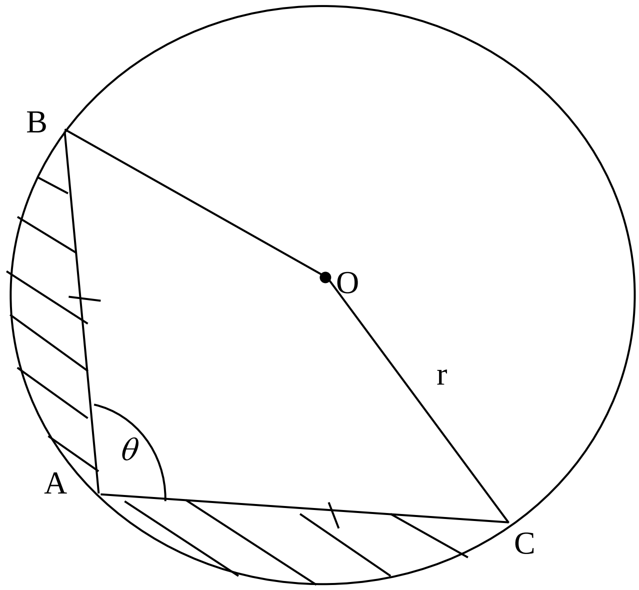
(i) Show that $\dfrac{dV}{dt} = k(A - V)$ [2]

(ii) Find the full volume i.e. $\lim\limits_{t \to \infty} V$ [1]

(iii) If one quarter of the vessel is filled in 10 minutes, what fraction is filled in the next 10 minutes? [2]

Question 13 continues on the next page.

b) AB and AC are two **equal** chords of a circle, whose centre is the point O and whose radius is r. The angle BAC is denoted by θ.



(i) Show that the triangles AOB and AOC are congruent. [2]

(ii) Prove that obtuse $\angle BOC = 2\pi - 2\theta$ [1]

(iii) Write down an expression for the area of each triangle in terms of r and θ. [1]

(iv) Find an expression for the area of minor sector BOC in terms of r and θ. [1]

(v) If the area bounded by the two chords AB, AC and the minor arc BC is equal to half of the area of the circle, show that

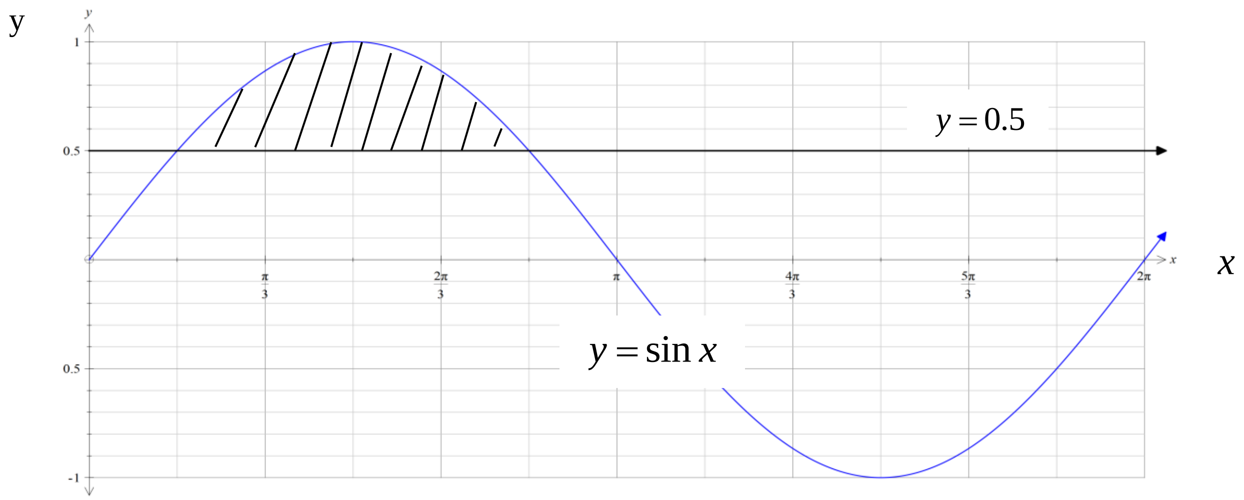
$$\theta + \sin\theta = \tfrac{\pi}{2}$$ [2]

(vi) Show that θ = 0.8 is an approximate solution to the equation in (v) and use one application of Newton's Method to find a better approximation, correct to 2 decimal places. [3]

Question 14 begins on the next page

Question 14 **Begin and label a new booklet.** **15 marks**

a)



Show that the volume generated when the area bounded by the curve $y = \sin x$ (for $0 < x < 2\pi$) and the line $y = 0.5$ is rotated about the x-axis is given

$$\frac{\pi}{4}\left[x - \sin 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

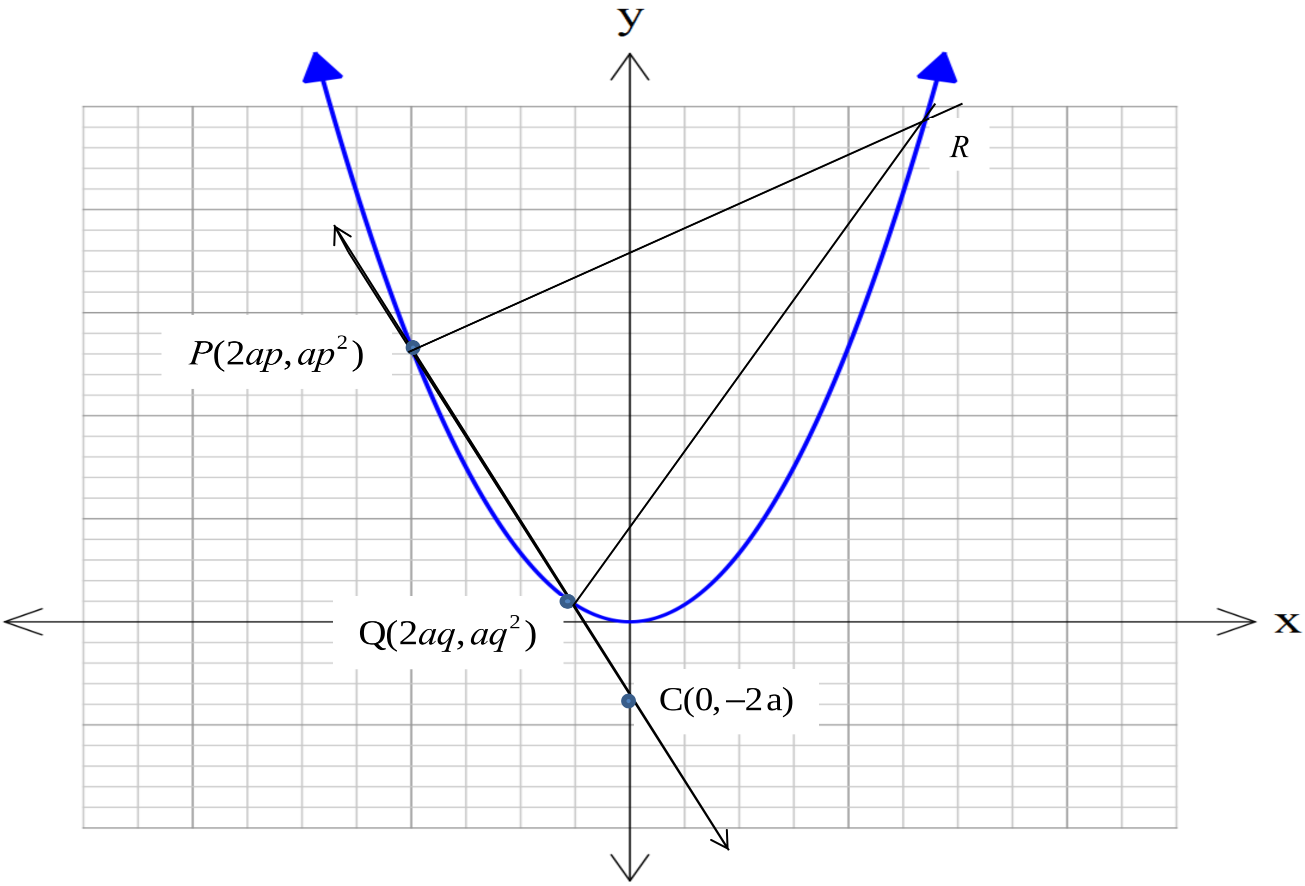
Do not evaluate this. [4]

b) A particle is projected from a horizontal plane at an angle of elevation of 30^0 with a speed of 100m/s. Taking g =10 m/s^2, find

(i) the equation of the trajectory (i.e. the Cartesian equation of the particles path). [3]

(ii) the range of the projectile and the time of flight [2]

Question 14 continues on the next page.



c) The normal to the parabola $x^2 = 4ay$ at points P($2ap, ap^2$) and Q($2aq, aq^2$) intersect at R. The chord PQ varies in such a way that for all positions of P and Q, the chord PQ when produced passes through the fixed point C(0, -2a).

(i) Find the equation of the chord PQ [1]

(ii) show that $pq=2$ [1]

(iii) Find the equations of the normal at both P and Q and hence find the coordinates of R [3]

(iv) Show that R lies on the parabola [1]

End of Exam

Turn over to find the table of standard integrals.

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\;dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\;dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\;dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\;dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Solutions to the Ascham 2014 Ext. 1 Trial

Section 1.

Q1

$$\tan\theta = \left|\frac{2--4}{1+2\times-4}\right|$$

$$= \left|\frac{6}{-7}\right|$$

$$= 6/7 \qquad D \checkmark$$

Q2.

$(-2,3) \quad (5,4)$

$-2:3$

$$\left(\frac{-10-6}{1}, \frac{9-8}{1}\right)$$

$$= (-16, 1) \qquad C \checkmark$$

Q3. B. ✓

Q4. A ✓

Q5.

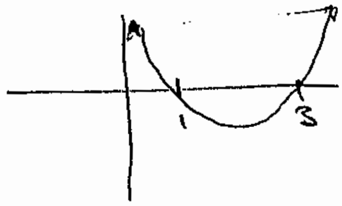
$$\frac{-2}{x-3} \le 1$$

$$-2(x-3) \le (x-3)^2$$

$$-2x+6 \le x^2-6x+9$$

$$0 \le x^2-4x+3$$

$$0 \le (x-3)(x-1)$$



$x \le 1$ or $x > 3$ D. ✓

$x \ne 3$ as $\frac{-2}{x-3}$ is then undefined.

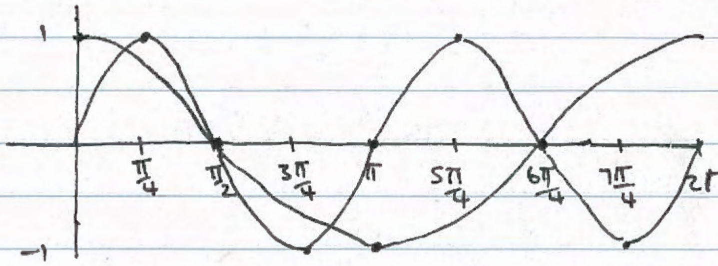
Q6. D ✓

Q7

$$\ddot{x} = -4(x+4) \qquad T = \frac{2\pi}{n} \qquad \text{centre } -4$$

$$\ddot{x} = -n^2(x--4) \qquad = \pi \qquad A. \checkmark$$

Q8



C ✓

Q9 D ✓

Q10 B ✓

Q11 a) i) $\frac{d}{dx} \cos^{-1}(3x) = \frac{-1}{\sqrt{1-9x^2}} \times 3$

$= \frac{-3}{\sqrt{1-9x^2}}$ ✓ $\frac{1}{1}$

ii) $\frac{d}{dx} \tan^2 3x$

$= 2\tan 3x \times \sec^2 3x \times 3$ ✓

$= 6\sec^2 3x \tan 3x$ ✓ $\frac{2}{2}$

b) $u = y+1$

$\therefore y = u-1$ when $y=3$ $u=4$

and $du = dy$ $y=0$ $u=1$] $\frac{1}{2}$

$\therefore \int_0^3 y\sqrt{y+1}\,dy = \int_1^4 (u-1)u^{1/2}\,du$ $\frac{1}{2}$

$= \int_1^4 u^{3/2} - u^{1/2}\,du$

$= \left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right]_1^4$ $\frac{1}{2}$

$= \left(\frac{64}{5} - \frac{16}{3}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)$

$= \frac{62}{5} - \frac{14}{3}$

$= 7\frac{11}{15}$ $\frac{1}{2}$ $\frac{2}{2}$

$\left(\frac{116}{15}\right)$

c). $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma(\alpha+\beta+\gamma)$ ✓½

$= -\frac{2}{2}\left(--\frac{1}{2}\right)$ ✓½

$= -1 \times \frac{1}{2}$ ✓½

$= -\frac{1}{2}.$ ✓½ 2/2

d) $x - \frac{\pi}{3} = \frac{\pi}{3} + k\pi$ ✓½

ie $x = \frac{2\pi}{3} + k\pi.$ ✓½ 1/1

e) Step 1. prove true for $n=1$

$5^n - 3^n = 5-3$
$= 2$ which is even ∴ true. ✓½

Step 2. Assume $5^k - 3^k$ is even for $n=k$
ie assume $5^k - 3^k = 2M$ for some integer M
To prove $5^{k+1} - 3^{k+1} = 2P$ for some integer P ✓

$5^{k+1} - 3^{k+1} = 5 \times 5^k - 3 \times 3^k$ ✓½

$= 3(5^k - 3^k) + 2 \times 5^k$ ✓½

$= 3 \times 2M + 2 \times 5^k$ (by assumption) ✓½

$= 2(3M + 5^k)$ where M is an integer and 5^k is too ✓½

∴ $5^{k+1} - 3^{k+1}$ is even as it has a common factor of 2. ✓½

Step 3 By steps 1 and 2 and the process of Mathematical induction the result is proven 4/4

f) $V = \frac{\pi}{3} x^2 (30 - x)$

$= 10\pi x^2 - \frac{\pi}{3} x^3$ ✓½

$\frac{dV}{dt} = 3\pi \qquad \frac{dx}{dt} = ? \quad \text{when} \quad x = 5$

$$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$$

$$V = 10\pi x^2 - \frac{\pi}{3} x^3$$

$$\therefore \frac{dV}{dx} = 20\pi x - \pi x^2$$ ✓½

$$\frac{dx}{dV} = \frac{1}{\pi x (20 - x)}$$ ✓½

$$\therefore \frac{dx}{dt} = \frac{1}{\cancel{\pi} x (20 - x)} \times 3\cancel{\pi}$$ ✓½

$$= \frac{3}{x(20 - x)}$$

$$= \frac{3}{5 \times 15}$$

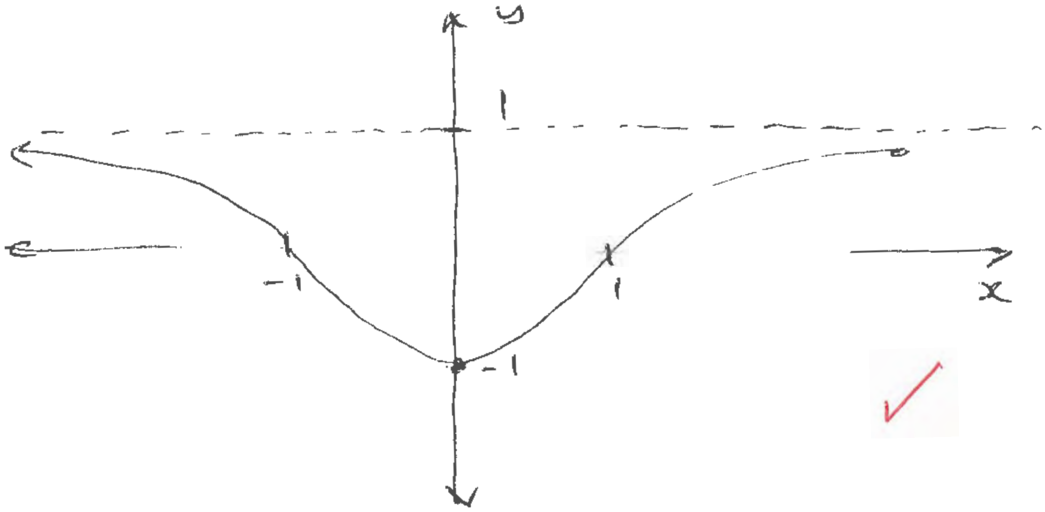
$$= \frac{1}{25}$$ ✓½

$\therefore$ depth is increasing at $\frac{1}{25}$ cm/min ✓½ /3

Question 12

a) i) $y = \dfrac{x^2-1}{x^2+1} = \dfrac{(x-1)(x+1)}{x^2+1}$

when $x = 0$ $y = -1$ ✓½ roots $x = 1$ or -1 ½



✓ 2/2

as $x \to \infty$ $y \to 1^-$

Function is even.

ii) $k = -1$ ✓ 1/1

iii) $-1 < k < 1$ ✓ 1/1

iv) $k < -1$ or $k \geq 1$ ✓ 1/1

b) $y = 3\sin^{-1}(1-x)$

Domain $-1 \leq 1-x \leq 1$

$-2 \leq -x \leq 0$

$2 \geq x \geq 0$ ✓

Range $-\frac{\pi}{2} \leq \sin^{-1}(1-x) \leq \pi/2$

$\therefore -\frac{3\pi}{2} \leq 3\sin^{-1}(1-x) \leq 3\pi/2$

ie $-\frac{3\pi}{2} \leq y \leq 3\pi/2$ ✓ 2/2

Q12 (b) ii)

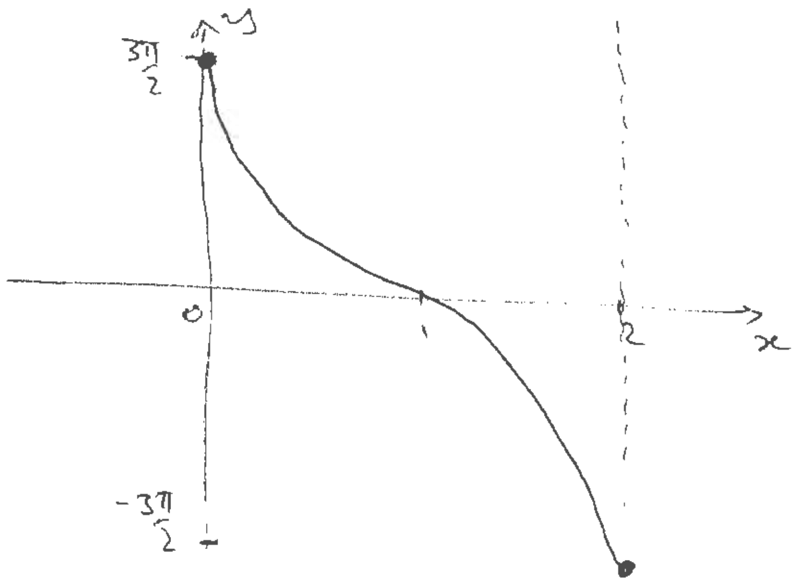
$$y' = 3 \frac{1}{\sqrt{1-(1-x)^2}} \times -1 \quad \tfrac{1}{2}$$

$$= \frac{-3}{\sqrt{1-(1-2x+x^2)}} \quad \tfrac{1}{2}$$

$$= \frac{-3}{\sqrt{2x-x^2}} \quad \tfrac{1}{2} \qquad \frac{-3}{1} = -3 \quad \tfrac{1}{2}$$

2/2

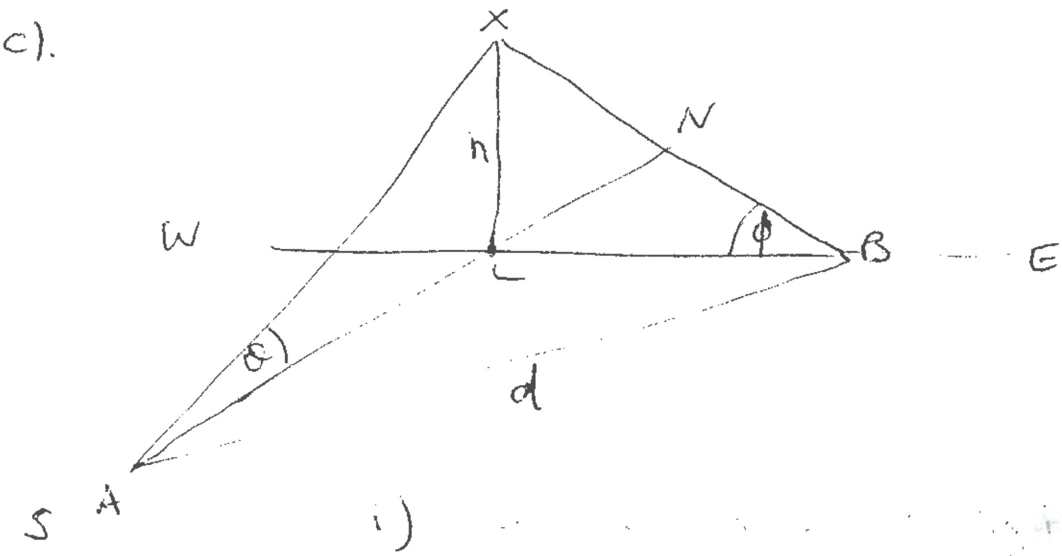
iii)



✓✓

2/2

c).



i)

$$\tan\theta = \frac{h}{AL}$$

$$\therefore AL = \frac{h}{\tan\theta} \quad \tfrac{1}{2}$$

Similarly $LB = \dfrac{h}{\tan\phi}$ ½

$\therefore d^2 = \left(\dfrac{h}{\tan\theta}\right)^2 + \left(\dfrac{h}{\tan\phi}\right)^2$ ½

$\therefore d^2 \tan^2\theta \tan^2\phi = h^2\tan^2\phi + h^2\tan^2\theta$ ½

$\therefore h^2 = \dfrac{d^2\tan^2\theta\tan^2\phi}{\tan^2\phi + \tan^2\theta}$ ½

$\therefore h = \dfrac{d\tan\theta\tan\phi}{\sqrt{\tan^2\phi + \tan^2\theta}}$ ½ $(h>0)$ 3/3

ii) $\therefore 115 = \dfrac{d\tan 18\tan 23°15'}{\sqrt{\tan^2 18 + \tan^2 23°15'}}$ ½

$d = 443.75228\ldots$

$= 443.75\text{m}$ (to 2dp) ½ 1/1

13a) i) $V = A(1 - e^{-kt})$

$dV/dt = A \times -e^{-kt} \times -k$ ½

but $1 - e^{-kt} = \frac{V}{A}$

$\therefore -e^{-kt} = \frac{V}{A} - 1$ ½

$\therefore dV/dt = A \times (\frac{V}{A} - 1) \times -k$ ½

$= -k(V - A)$ ½

$= k(A - V)$ ← This line is given. $\frac{2}{2}$

ii) $\lim_{t \to \infty} V = \lim_{t \to \infty} A(1 - e^{-kt})$

$= A$ ✓ $\frac{1}{1}$

iii) at $t = 0$ $V = A(1 - e^{0})$

$= 0$

Full volume is A

$\therefore \frac{1}{4}A = A(1 - e^{-10k})$ ½

ie $e^{-10k} = 3/4$

$-10k = \ln 3/4$

$k = -\frac{1}{10} \ln 3/4$ ½

At $t = 20$

$V = A(1 - e^{-20k})$

$= A(1 - e^{-20 \times -\frac{1}{10} \ln 3/4})$ ½

$= A(1 - e^{2\ln 3/4})$

$= A(1 - e^{\ln 9/16})$

$= A(1 - 9/16)$

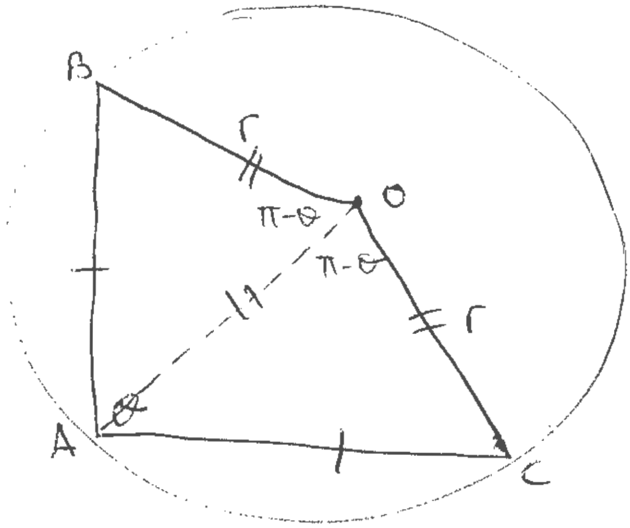
$= \frac{7}{16}A$ $\frac{2}{2}$

$\therefore$ fraction filled in next 10 minutes $= \frac{7}{16}A - \frac{1}{4}A$

$= \frac{3}{16}A$

ie $3/16$ are filled in the next 10 minutes ½

b) i)



In Δs AOB and AOC ✓½

$BO = AO = CO = r$ (radii) ✓½

$BA = AC$ (given) ✓½

$\therefore \triangle AOB \equiv \triangle AOC$ (SSS) ✓½ $\frac{2}{2}$

ii) Reflex $\angle BOC = 2\theta$ (angle at the centre is twice the angle at the circumference, standing on the same arc) ✓½

$\therefore$ Obtuse $\angle BOC = 2\pi - 2\theta$. ← Given. ✓½ $\frac{1}{1}$

iii) Area of each $\Delta = \frac{1}{2}r^2\sin(\pi-\theta)$ ✓ $\frac{1}{1}$

iv) Area of minor sector $= \frac{2\pi-2\theta}{2\pi}\pi r^2$ ✓½

$= \frac{\pi-\theta}{\pi}\times\pi r^2$

$= (\pi-\theta)r^2$. ✓½ $\frac{1}{1}$

v) Shaded Area $= (\pi-\theta)r^2 - 2\times\frac{1}{2}r^2\sin(\pi-\theta)$ ✓½

$\therefore \frac{1}{2}\pi r^2 = (\pi-\theta)r^2 - r^2\sin(\pi-\theta)$ ✓½

$$\tfrac{1}{2}\pi r^2 = \pi r^2 - \theta r^2 - r^2\sin(\pi-\theta)$$

$$0 = \tfrac{1}{2}\pi - \theta - \sin(\pi-\theta)$$

ie $\theta + \sin(\pi-\theta) = \pi/2$ ½

ie $\theta + \sin\theta = \pi/2$ ← Given since $\sin(\pi-\theta) = \sin\theta$ ½

2/2

vi) let $\theta = 0.8$

let $y = \theta + \sin\theta - \pi/2$

$\therefore y = 0.8 + \sin 0.8 - \pi/2$ ½

$= -0.05344\ldots$ ½ 1/1

≈ 0

$\therefore$ There is a root close to $\theta = 0.8$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x) = \theta + \sin\theta - \frac{\pi}{2}$

$f'(x) = 1 + \cos\theta$ ½

let $x_0 = 0.8$

then $x_1 = 0.8 - \dfrac{0.8 + \sin 0.8 - \pi/2}{1 + \cos 0.8}$ ✓

$= 0.831496\ldots$

$= 0.83$ (to 2dp) ½ 2/2

Q14 a) Volume $= \int \pi y^2 \, dx$

$\therefore$ required volume $= \pi \int_{\pi/6}^{5\pi/6} y_1^2 - y_2^2 \, dx$ ½

$= \pi \int_{\pi/6}^{5\pi/6} \sin^2 x - (0.5)^2 \, dx$ ½

$\sin x = 0.5$

$x = \pi/6$ or $5\pi/6$ ½

$= \pi \int_{\pi/6}^{5\pi/6} \sin^2 x - \frac{1}{4} \, dx$ ½

$= \pi \int_{\pi/6}^{5\pi/6} \frac{1}{2}(1 - \cos 2x) - \frac{1}{4} \, dx$ ½

$\cos 2\theta = 1 - 2\sin^2\theta$

$\therefore \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ ½

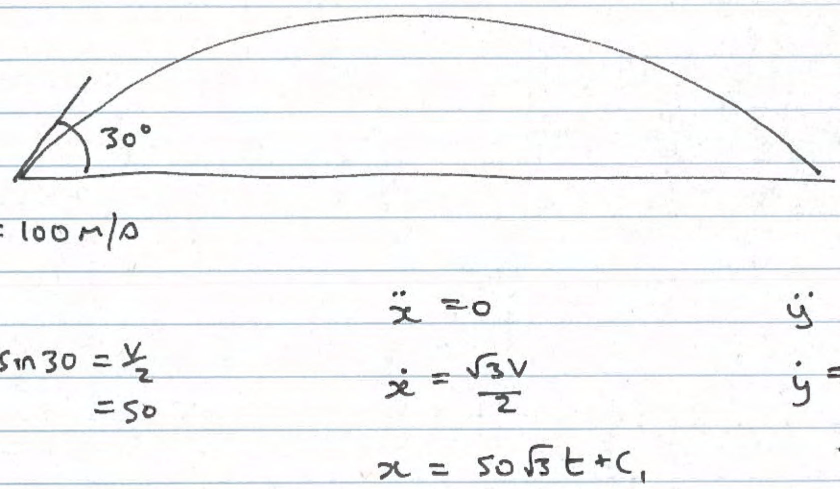
$= \pi \int_{\pi/6}^{5\pi/6} \frac{1}{4} - \frac{1}{2}\cos 2x \, dx$ ½

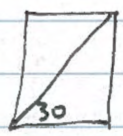
$= \pi \left[\frac{1}{4}x - \frac{1}{2} \times \frac{\sin 2x}{2} \right]_{\pi/6}^{5\pi/6}$ ½

$= \frac{\pi}{4} \left[x - \sin 2x \right]_{\pi/6}^{5\pi/6}$ shown.

4/4.

b)





$V\sin 30 = \frac{V}{2}$
$= 50$

$V\cos 30$
$= \frac{\sqrt{3}V}{2}$
$= 50\sqrt{3}$

$\ddot{x} = 0$

$\dot{x} = \frac{\sqrt{3}V}{2}$

$x = 50\sqrt{3}t + C_1$ ½

$\ddot{y} = -10$

$\dot{y} = -10t + C$

$= -10t + \frac{V}{2}$

$= -10t + 50$

$y = -5t^2 + 50t + C_2$ ½

at $t = 0, x = 0$ $y = 0$

$\therefore C_1 = 0$ and $C_2 = 0$

$\therefore x = 50\sqrt{3}t$ $y = -5t^2 + 50t$ ½

$\therefore t = \frac{x}{50\sqrt{3}}$ ½

$\therefore y = -5\left(\frac{x^2}{2500 \times 3}\right) + 50 \times \frac{x}{50\sqrt{3}}$ ½

$$y = \frac{-x^2}{1500} + \frac{x}{\sqrt{3}} \quad ✓\tfrac{1}{2}$$

$$\left(= \frac{-x^2}{1500} + \frac{\sqrt{3}\,x}{3} \right)$$

3/3

ii) when $y = 0$ $\quad 0 = -5t^2 + 50t$ ✓½

$= 5t(10 - t)$ ✓½ $\therefore t = 0$ or 10

$\therefore$ particle returns to ground at $t = 10$ sec. ✓½

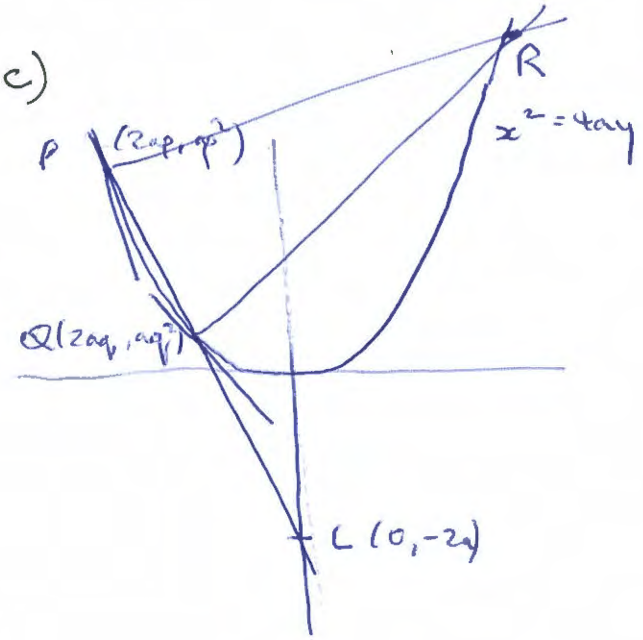
At this time $\quad x = 50\sqrt{3}\,t$

$= 500\sqrt{3}$ m

$\therefore$ the range is $500\sqrt{3}$ m. ✓½

2/2

c)



i) chord PQ

$$y - ap^2 = \frac{ap^2 - aq^2}{2a(p-q)}(x - 2ap) \quad ✓\tfrac{1}{2}$$

$$y - ap^2 = \frac{\cancel{a}\cancel{(p-q)}(p+q)}{2\cancel{a}\cancel{(p-q)}}(x - 2ap)$$

$$y - ap^2 = \frac{p+q}{2}(x - 2ap) \quad ✓\tfrac{1}{2} \quad \tfrac{1}{1}$$

ii) $(0,-2a)$ satisfies

ie $$-2a - ap^2 = \frac{p+q}{\cancel{2}} \times -\cancel{2}ap \quad ✓\tfrac{1}{2}$$

$$-2a - ap^2 = -ap(p+q)$$

$$-2a - \cancel{ap^2} = -\cancel{ap^2} - apq$$

$$-2\cancel{a} = -\cancel{a}pq$$

$$pq = 2. \quad ✓\tfrac{1}{2} \quad \tfrac{1}{1}$$

iii) normal at P

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$yp - ap^3 = -x + 2ap$$

normal at Q $$yq - aq^3 = -x + 2aq \quad ✓$$

$$y(p-q) - a(p^3 - q^3) = 2a(p-q)$$

$$y - a(p^2 + pq + q^2) = 2a$$

$$y = 2a + a(p^2 + pq + q^2)$$

$$y = a(p^2 + pq + 2 + q^2)$$
$$= a(p+q)^2 \quad ✓$$

$$x = ap^3 - yp + 2ap$$
$$= ap^3 - pa(p+q)^2 + 2ap$$
$$= ap^3 - ap(p^2 + 2pq + q^2) + 2ap$$
$$= -2ap^2q - apq^2 + 2ap$$
$$= -4ap - 2aq + 2ap$$
$$= -2ap - 2aq$$
$$= -2a(p+q) \quad ✓$$

$\therefore R$ is $(-2a(p+q),\ a(p+q)^2)$

3/3

iv) $x^2 = 4a^2(p+q)^2$ ✓½

$= 4ay$. ✓½

1/1